Mixed-Integer Multiobjective Process Planning under Uncertainty

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This paper presents a methodology for addressing investment planning in the process industry using a mixed-integer multiobjective approach. The classic single-objective MILP stochastic model is treated as a multiobjective programming problem by the use of multiparametric decomposition. To ensure computation of the entire efficient frontier, use of the augmented Tchebycheff algorithm is proposed. This allows for the decision making to be based on the suggested solutions and their neighbor solutions, a feature that other stochastic programming models fail to capture. An iterative procedure is proposed to help the decision maker visualize the efficient solutions in the multidimensional space and facilitate the assessment of the economical risk of the project.

Introduction

Financial planning in the chemical industry is important in considerations of the potential benefits of using new resources in conjunction with existing ones. Planning is necessary to assess the potential impacts of changes on demand, prices, technology, capital, markets, and competition. Jimenez and Rudd[1] introduced a recursive mixed-integer linear programming (MILP) technique to solve this problem, whereas Sahinidis et al.[2] proposed the use of MILP techniques that can handle large-scale networks. When analyzing this deterministic formulation, Ahmed and Sahinidis[3] proved that the problem under study is NP-hard. Judging from this result, the authors presented an asymptotically optimal approximation scheme for this problem. A similar approach was taken by Liu and Sahinidis[4] in applying analytical investigations to the development of an asymptotically optimal heuristic. Liu et al.[5] developed algorithms using an entire continuous model that allows for more general objective functions. An efficient cutting plane algorithm that includes more variables and constraints but is more robust and faster than conventional approaches was developed by Liu and Sahinidis.[6]

Maximization of the expected net present value is the single objective when the standard two-stage stochastic programming formulation is used to model process planning under uncertainty.[7−9] Because fixed probabilities are utilized, the opportunity for exploring other efficient alternatives is lost. Consideration of these alternatives allows for the development of direct methods for the evaluation of the economical risk involved in a project. Ahmed and Sahinidis[10] recognized a potential limitation of the stochastic scenario-based model: it does not include any conditions on the variability of the costs incurred in the second stage. Consequently, they extended the stochastic programming formulation by using a variability index to account for robustness of these costs. However, no explicit definition of risk was presented, and its assessment was left to the decision maker. The economical risk involved in process planning stems from the uncertainty in the estimation of the net present value parameters. This issue was discussed by Applequist et al.,[11] who attempted to incorporate risk explicitly by using the risk premium concept. Variability studies can be conducted using the upper partial mean[10] to establish solutions with acceptable risk.

All this work notwithstanding, we believe that there is one element of the whole procedure that has been overlooked: the decision maker and his/her input. Quite clearly, all of the aforementioned works do not assess risk in a satisfactory manner and commit to find one solution, therefore relying on an a priori input by a decision maker. Decision making in reality is a much more interactive process. The decision maker sometimes wants to "get a feeling" of the suggested solutions and their neighbor solutions, so that he/she can alter the parameters slightly and look for an improved objective function. A tool for such interactive procedure is attempted in this article. Previous work by Shimizu and Takamatsu[12] considered the input of the decision maker by the use of a stepwise and subjective judgment process. The proposed interactive goal programming algorithm was the first multiobjective approach to the planning problem, even though no uncertainty was considered.

In the present work, the single-objective MILP stochastic programming model by Liu and Sahinidis[7] is treated as a multiobjective programming problem by using multiparametric decomposition.[13] The point-estimate weighted-sums approach[14] is one of the possible methods that can be used to obtain the set of efficient solutions. This method makes use of the probabilities of each scenario to weight the respective objectives. However, because of the mixed-integer linear nature of the problem at hand, only supported efficient solutions are found by this method. Reformulation of the problem as an augmented weighted Tchebycheff program (similar to the approach of Pinto and Rustem[15]) makes possible the computation of all efficient solutions. The objective is to scan the efficient frontier to provide the decision maker with the freedom to select a solution by specifying aspiration levels. Different methods of specifying these aspiration levels are possible. This paper presents an iterative procedure based on the use of lower bounds for the net present value that facilitates the assessment of the economical risk of a project. An explanation of terms from multiobjective optimization can be found in the Appendix.

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10.1021/ie010530j CCC: $22.00
Published on Web 07/11/2002

Single-Objective Multiperiod MILP Model

The model first introduced by Sahinidis et al.[2] for the optimal selection and expansion of processes given time-varying forecasts for the demands and prices of chemicals over a long-range horizon is reviewed. Consider a network of NP chemical processes interconnected in a finite number of ways, including NC chemicals that are fed to and/or obtained from the processes. This network accounts for existing as well as potential new processes and chemicals. Figure 1shows the example of a network consisting of three chemical processes and three chemicals.

Also, consider a finite number of NT periods during which prices of and demands for chemicals from a set of NM markets and investments and operating cost of the processes can vary. Therefore, a multiperiod MILP problem that maximizes the net present value for the given network and the given forecasts for prices and demands of chemicals as well as investment and operating costs can be formulated. The following is determined to maximize the net present value of the project: (a) capacity expansion and shut-down policy for existing processes, (b) selection of new processes and their capacity expansion policy, (c) production profiles, and (d) sales and purchases of chemicals at each time period.

The single-objective deterministic problem is

Model SODP

$$\max \sum_{t=1}^{NT}\sum_{l=1}^{NM}\sum_{j=1}^{NC}(\gamma_{jlt}S_{jlt} - \Gamma_{jlt}P_{jlt}) - \sum_{t=1}^{NT}\sum_{i=1}^{NP}\delta_{it}W_{it} - \sum_{i=1}^{NP}\sum_{t=1}^{NT}(\alpha_{it}E_{it} + \beta_{it}Y_{it}) \quad (1)$$

s.t.

$$\left.\begin{aligned} Y_{it}E_{it}^{L} \le E_{it} \le Y_{it}E_{it}^{L} \\ Q_{it} = Q_{i(t-1)} + E_{it} \end{aligned}\right\} \quad i = 1, ..., NP;\ t = 1, ..., NT \quad (2)$$

$$\sum_{t=1}^{NT} Y_{ij} \le NEXP(i) \qquad i \in I' \subseteq \{1, 2, ..., NP\} \quad (3)$$

$$\sum_{i=1}^{NP}(\alpha_{it}E_{it} + \beta_{it}Y_{it}) \le CI(t) \qquad t \in T' \subseteq \{1, 2, ..., NT\} \quad (4)$$

$$\left.\begin{aligned} & W_{it} \le Q_{it} && i = 1, ..., NP \\ & \sum_{l=1}^{NM}P_{jlt} + \sum_{i=1}^{NP}\eta_{ij}W_{it} = \\ & \quad \sum_{l=1}^{NM}S_{jlt} + \sum_{i=1}^{NP}\mu_{ij}W_{it} && j = 1, ..., NC \\ & \left.\begin{aligned} \alpha_{jlt}^{L} \le P_{jlt} \le \alpha_{jlt}^{U} \\ d_{jlt}^{L} \le S_{jlt} \le d_{jlt}^{U} \end{aligned}\right\} && j = 1, ..., NC;\ l = 1, ..., NM \end{aligned}\right\} t = 1, ..., NT \quad (5)$$

$$\begin{aligned} & Y_{it} \in \{0,1\} \qquad i = 1, ..., NP;\ t = 1, ..., NT \\ & E_{it}, Q_{it}, W_{it}, P_{jlt}, S_{jlt} \ge 0 \qquad \forall\ i, j, l, t \end{aligned} \quad (6)$$

The objective function to be maximized (eq 1) is the net present value of the planning problem over the given horizon of NT periods. It is composed of the difference between the variables representing the total sales S_{jlt} and total purchases P_{jlt} of chemicals at their respective market prices γ_{jlt} and Γ_{jlt}, minus the total operating costs W_{it} with unit operating cost coefficient δ_{it}. In addition, the investment cost, composed of a variable term representing the size of the expansion E_{it} (with unit cost α_{it}) and a fixed term represented by the binary variable Y_{it} (with unit cost β_{it}), is subtracted.

Constraints that guarantee lower and upper bounds in the capacity expansion and compute the total capacity available are added by eqs 2. Limits on the number of expansions of processes and on the capital available are expressed by inequalities 3 and 4, respectively. Equations 5 ensure material balance, limit the operating level of a process not to exceed the installed capacity, and express lower and upper bounds for raw materials and products. Finally, the definitions of the variables are provided in expressions 6.

Single-Objective Stochastic Programming Model

The following single-objective model presented by Liu and Sahinidis[7] considers process planning from the stochastic point of view. This model is an extension of the deterministic mixed-integer linear programming formulation SODP introduced in the previous section. It is based in multiple scenarios, each of which has fixed parameters. The model, therefore, does not rely on a continuous distributions of parameters.

Model SOSP

$$\max \sum_{s=1}^{NS}p_s\Big(\sum_{t=1}^{NT}\sum_{l=1}^{NM}\sum_{j=1}^{NC}(\gamma_{jlts}S_{jlts} - \Gamma_{jlts}P_{jlts}) - \sum_{t=1}^{NT}\sum_{i=1}^{NP}\delta_{its}W_{its}\Big) - \sum_{i=1}^{NP}\sum_{t=1}^{NT}(\alpha_{it}E_{it} + \beta_{it}Y_{it})$$

s.t.

constraints 2−4

$$\left.\begin{aligned} & W_{its} \le Q_{it} && i = 1, ..., NP \\ & \sum_{l=1}^{NM}P_{jlts} + \sum_{i=1}^{NP}\eta_{ij}W_{its} = \\ & \quad \sum_{l=1}^{NM}S_{jlts} + \sum_{i=1}^{NP}\mu_{ij}W_{its} && j = 1, ..., NC \\ & \left.\begin{aligned} \alpha_{jlts}^{L} \le P_{jlts} \le \alpha_{jlts}^{U} \\ d_{jlts}^{L} \le S_{jlts} \le d_{jlts}^{U} \end{aligned}\right\} && j = 1, ..., NC;\ l = 1, ..., NM \end{aligned}\right\} \begin{aligned} t = 1, ..., NT \\ s = 1, ..., NS \end{aligned} \quad (8)$$

$$\begin{aligned} & Y_{it} \in \{0,1\} \qquad i = 1, ..., NP;\ t = 1, ..., NT \\ & E_{it}, Q_{it}, W_{its}, P_{jlts}, S_{jlts} \ge 0 \qquad \forall\ i, j, l, t, s \end{aligned} \quad (9)$$

In this model, the objective function maximizes the expected net present value (NPV) over two stages of the capacity expansion project. The first-stage decisions are the investment cost, which is represented by a variable term that is proportional to the capacity expansion, and a fixed term that is included using a binary variable. The second-stage decisions are described by finitely many and mutually exclusive scenarios s with corresponding probabilities p_s that are independent of the

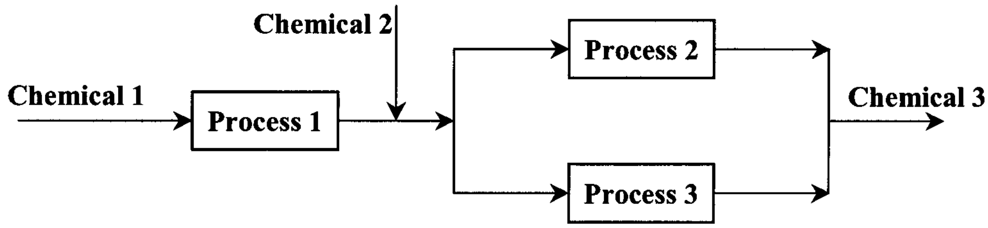

Figure 1. Example of a network of chemical processes.

first-stage decisions. These decisions are the expectation of the sales revenues and the expectation of the second-stage operating costs. If one has probability distributions or equivalent information, one way of constructing these scenarios is to discretize these distributions, that is, to take selected values of prices and costs and compute the combined probability p_s.

Multiparametric Decomposition

By using the probabilistic scenario analysis approach presented in the previous section and adding the multiparametric decomposition methodology proposed by Trafalis et al.,[13] model SOSP can be reformulated as a multiobjective MILP problem.

Model MOSP

$$\max\ p_1\Big[\sum_{t=1}^{NT}\sum_{l=1}^{NM}\sum_{j=1}^{NC}(\gamma_{jlt1}S_{jlt1} - \Gamma_{jlt1}P_{jlt1}) - \sum_{t=1}^{NT}\sum_{i=1}^{NP}\delta_{it1}W_{it1} - \sum_{i=1}^{NP}\sum_{t=1}^{NT}(\alpha_{it}E_{it} + \beta_{it}Y_{it})\Big]$$

$$\vdots$$

$$\max\ p_{NS}\Big[\sum_{t=1}^{NT}\sum_{l=1}^{NM}\sum_{j=1}^{NC}(\gamma_{jltNS}S_{jltNS} - \Gamma_{jltNS}P_{jltNS}) - \sum_{t=1}^{NT}\sum_{i=1}^{NP}\delta_{itNS}W_{itNS} - \sum_{i=1}^{NP}\sum_{t=1}^{NT}(\alpha_{it}E_{it} + \beta_{it}Y_{it})\Big] \quad (10)$$

s.t.

constraints 2−4

constraints 8 and 9

This formulation can be seen as a point-estimate weighted-sums approach[14] with the weights represented by the probabilities. If the assessment of the values of the probabilities is left to the decision maker (DM), the problem becomes a mixed-integer linear programming problem, and it is guaranteed to obtain an efficient proper solution for strictly positive values of p_s.

Iterative Augmented Weighted Tchebycheff Program

Because of the mixed-integer nature of the planning problem, difficulties are sometimes encountered in solving the parametric problem MOSP by the weighted-sums approach. Only supported efficient solutions can be computed, and the complete scanning of the efficient frontier is not possible when unsupported criterion vectors or discontinuities in the efficient frontier exist. Therefore, the use of an augmented weighted Tchebycheff program[15] is proposed to overcome those difficulties and allow for the complete scanning of the efficient frontier.

Table 1. Fixed and Variable Investment Coefficients for Example 1

process/ period	β_{it} ($\times 10^5$)			α_{it} ($\times 10^5$)		
	$t = 1$	$t = 2$	$t = 3$	$t = 1$	$t = 2$	$t = 3$
1	206	257	399	9.25	8.03	4.9
2	484	510	300	5.18	10.97	11.2
3	409	547	403	8.46	5.97	7.64
4	294	532	550	8.16	4.18	7.68
5	508	496	401	8.41	5.35	4.42
6	334	396	568	10.36	6.39	9.12

Consider the criterion vector definition for this case as the value of the NPV. This vector will be inside the feasible region if all of the constraints already introduced for the multiperiod MILP problem are verified. Therefore, the criterion vectors are

$$\mathbf{z}_s = \sum_{t=1}^{NT}\sum_{l=1}^{NM}\sum_{j=1}^{NC}(\gamma_{jlts}S_{jlts} - \Gamma_{jlts}P_{jlts}) - \sum_{t=1}^{NT}\sum_{i=1}^{NP}\delta_{its}W_{its} - \sum_{i=1}^{NP}\sum_{t=1}^{NT}(\alpha_{it}E_{it} + \beta_{it}Y_{it}) \quad (11)$$

Thus, the following augmented weighted Tchebycheff program can be used to scan all efficient solutions for any network structure realization between those of the scenarios under consideration

Model AWTP

$$\min\ (\alpha - \sum_{s=1}^{NS}\rho_s z_s) \quad (12)$$

s.t.

$$\alpha \geq \lambda_s(z_s^{**} - z_s) \qquad s = 1, ..., NS \quad (13)$$

constraints 2−4

constraints 8 and 9

Example 1

To illustrate the concepts presented, example 8 from Liu and Sahinidis[7] is considered. This example consists of two scenarios with data given in Table 1 and network structure shown in Figure 2. The first scenario is the deterministic example taken from Liu and Sahinidis.[8] Both examples have no process initially installed. Three time periods of 2, 2, and 3 years length are used. The maximum number of expansions is 2, and the capital limits are \$892 × 10^5, \$446 × 10^5, and \$975 × 10^5, respectively. The upper bound on capacity expansion is 100 kton/year for all processes in all periods. Tables 1−3, taken from Liu and Sahinidis,[8] give the rest of the

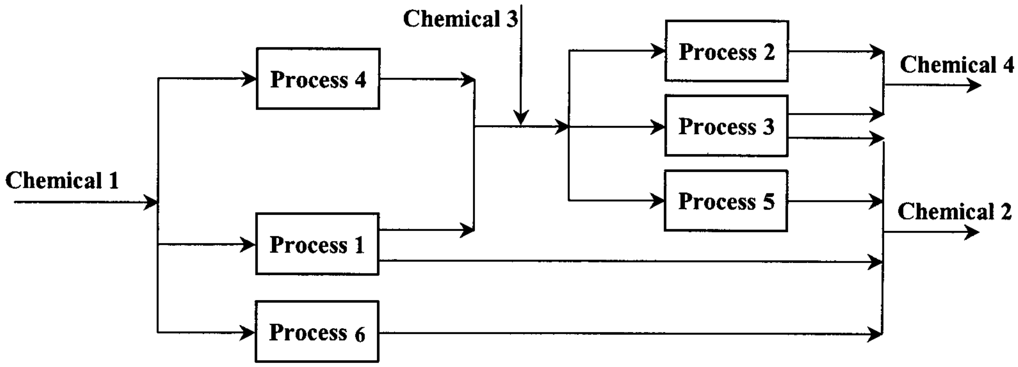

Figure 2. Network for example 1.

Table 2. Process Availabilities and Demands for Example 1

chemical/ period	price ($\times 10^2$/ton)			availability (α^{U}_{jlts}) or demand (d^{U}_{jlts}) (kton/year)		
	$t = 1$	$t = 2$	$t = 3$	$t = 1$	$t = 2$	$t = 3$
1	37.54	28.40	25.87	53	55	36
2	51.36	23.77	34.71	52	73	60
3	29.05	22.69	34.71	50	58	34
4	42.04	45.64	55.48	61	72	85

Table 3. Operational Expenses and Mass Balance Coefficients for Example 1

process/period or chemical	δ_{it} (\$/ton)			μ_{ij} or (η_{ij})			
	$t = 1$	$t = 2$	$t = 3$	$C = 1$	$C = 2$	$C = 3$	$C = 4$
1	20	30	20	1.11	(0.59)	(1)	
2	30	30	30			0.51	(1)
3	60	50	50		(1)	0.74	(0.87)
4	30	40	30	0.57		(1)	
5	60	50	40	0.61	(1)	0.58	
6	60	60	40	0.93	(1)		

Table 4. Results for Deterministic Models for Example 1

	scenario 1			scenario 2		
	$Q_{1,t}/W_{1,t}$ [a]	$Q_{2,t}/W_{2,t}$ [b]	$Q_{3,t}/W_{3,t}$ [c]	$Q_{1,t}/W_{1,t}$ [a]	$Q_{2,t}/W_{2,t}$ [b]	$Q_{3,t}/W_{3,t}$ [c]
$t = 1$			57.1/52.0			57.1/57.1
$t = 2$	23.5/23.5		57.1/57.1	23.5/23.5		57.1/57.1
$t = 3$	23.5/23.5	44.9/44.9	57.1/46.1	23.5/23.5	46.6/46.6	57.1/57.1

[a] Process 1. [b] Process 2. [c] Process 3.

example data for scenario 1. The second scenario uses values of all of the parameters that are 25% higher than those used in scenario 1. The first scenario was assumed to have a probability of 0.75, and the second scenario a probability of 0.25.

For the purpose of our study, no fixed value of the probabilities is considered, and the multiparametric decomposition results in two objectives, one for each scenario. If each scenario is solved separately using the deterministic model SODP, the components of the ideal criterion vector (maximum net present values for each scenario) are obtained. For this example, these values are $z_1^* = \$9{,}293.19 \times 10^5$ and $z_2^* = \$13{,}490.50 \times 10^5$.

Table 4 shows the results of the deterministic model for each scenario, and Figure 3 depicts the corresponding network. The only difference between the solutions for the structures of the two scenarios at the beginning of each period is a higher value of the installed capacity for process 2 during the third period. Also, note that scenario 1 makes partial use of the installed capacity of process 3 during the first and third periods.

The components of the nadir criterion vector can be estimated by forming the payoff table and finding the minimum values for each of the columns. Because this example consists of only two scenarios, the components of the nadir criterion vector are directly computed by maximizing the net present value for one scenario (the scenario determining the component of the vector) while fixing the values of the capacities at the values obtained when solving for the other scenario. These components are $z_1^{nd} = \$\ 9{,}273.45 \times 10^5$ and $z_2^{nd} = \$\ 13{,}427.66 \times 10^5$.

Solving the augmented weighted Tchebycheff program (AWTP) for a finite number of λ values makes possible the construction of the efficient frontier. Table 5 shows the net present values for the scenarios at an equally space finite number of λ values. The values from Table 5 were used to construct the plot shown in Figure 4, where the set of nondominated criterion vectors (efficient frontier if it is mapped to the decision space) is the entire diagonal line. The locations of the ideal and nadir criterion vectors are also shown in this figure.

The augmented term is required to avoid computation of dominated criterion vectors. From Figure 4, it is evident that, without the use of the augmented term, any criterion vector of the horizontal and vertical edges can be obtained by using the Tchebycheff metric. Therefore, by using a small value for ρ_s (e.g., 0.000 01), it is ensured that only the extreme points are obtained.

Scalarization. An analysis of Figure 4 also reveals a higher concentration of points on one of the sides of the diagonal. To obtain a uniform distribution of points, a scalarization of the objectives is conducted. The new version of the augmented weighted Tchebycheff program is therefore

Scalarized Model AWTP

$$\min \left[\alpha - \sum_{s=1}^{NS} \rho_s \left(\frac{z_s}{(z_s^* - z_s^{nd})}\right)\right]$$

s.t.

$$\alpha \geq \lambda_s \left(\frac{z_s^* - z_s}{z_s^* - z_s^{nd}}\right) \qquad s = 1, ..., NS$$

constraints 2−4

constraints 8 and 9

In this model, the difference between the ideal and nadir criterion vectors $(z_s^* - z_s^{nd})$ is used to normalize the objectives.

Example 1 (Continued). Table 6 shows the new values for the NPVs using the normalized augmented

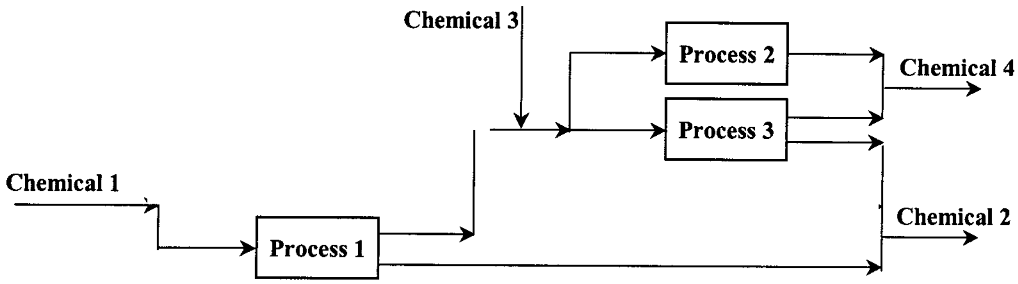

Figure 3. Solution network for example 1.

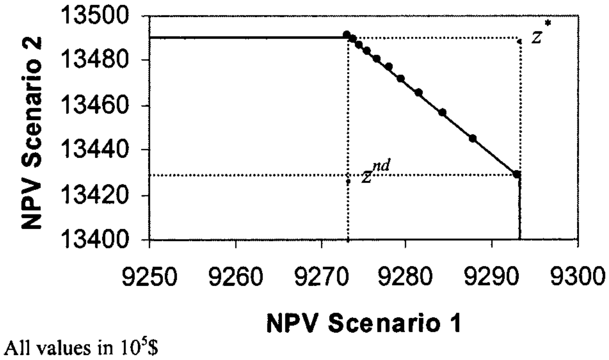

Figure 4. Multicriteria solution for example 1.

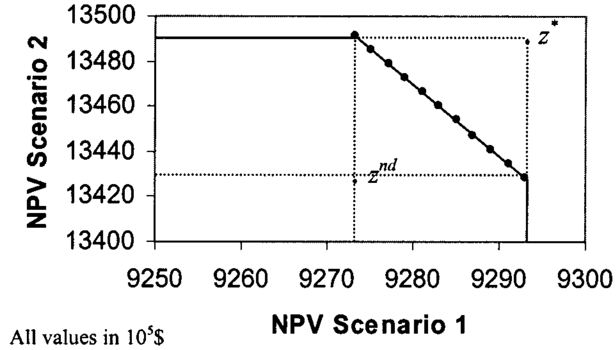

Figure 5. Scalarized multicriteria solution for example 1.

Table 5. NPVs for AWTP Model

λ_1/λ_2	z_1 ($\times 10^5$ \$)	z_2 ($\times 10^5$ \$)
1.0/0.0	9,293.19	13,427.66
0.9/0.1	9,288.10	13,443.87
0.8/0.2	9,284.48	13,455.37
0.7/0.3	9,281.82	13,463.84
0.6/0.4	9,279.78	13,470.34
0.5/0.5	9,278.17	13,475.48
0.4/0.6	9,276.86	13,479.64
0.3/0.7	9,275.78	13,483.09
0.2/0.8	9,274.86	13,485.99
0.1/0.9	9,274.09	13,488.46
0.0/1.0	9,273.45	13,490.50

Table 6. NPVs for Scalarized AWTP Model

λ_1/λ_2	z_1 ($\times 10^5$ \$)	z_2 ($\times 10^5$ \$)
1.0/0.0	9,293.19	13,427.66
0.9/0.1	9,291.30	13,433.69
0.8/0.2	9,289.31	13,440.03
0.7/0.3	9,287.31	13,446.37
0.6/0.4	9,285.33	13,452.70
0.5/0.5	9,283.34	13,459.03
0.4/0.6	9,281.35	13,465.35
0.3/0.7	9,279.36	13,471.67
0.2/0.8	9,277.38	13,477.98
0.1/0.9	9,275.40	13,484.29
0.0/1.0	9,273.45	13,490.50

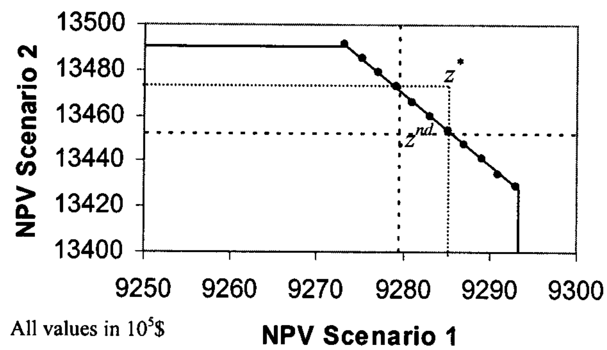

Figure 6. Aspiration levels for the NPVs.

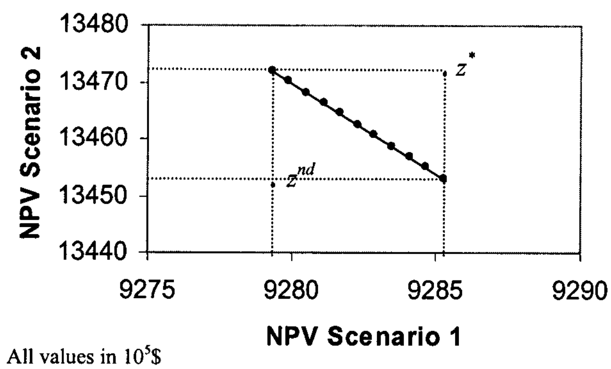

Figure 7. Region bounded by the aspiration levels.

weighted Tchebycheff program. Figure 5 shows the plot of the feasible region with a uniform distribution of points.

Iterative Procedure. Consider now that a minimum value for the net present values for the realization of each scenario is introduced. Then, the set of nondominated criterion vectors is reduced, and new values for the components of the ideal criterion vectors are calculated. The values of the components of the nadir criterion vector are the established bounds because they limit from below the realization of the scenarios. The decision maker is the one who narrows these narrower aspiration levels. The iterative procedure can be summarized as follows:

Step 1. Determine ideal and nadir criterion vectors. Ask the decision maker to give aspiration levels (upper and lower bounds on profit).

Step 2. Compute compromise solutions by solving the augmented weighted Tchebycheff program using the aspiration levels requested.

Step 3. Present the solution to the decision maker. If the decision maker is satisfied, stop. Otherwise, ask for new aspiration levels

Example 1 (Continued). Consider that the bounds for each scenario are a_1^{I} = \$ 9,279.36 and a_2^{I} = \$13,452.70. These values also correspond to components

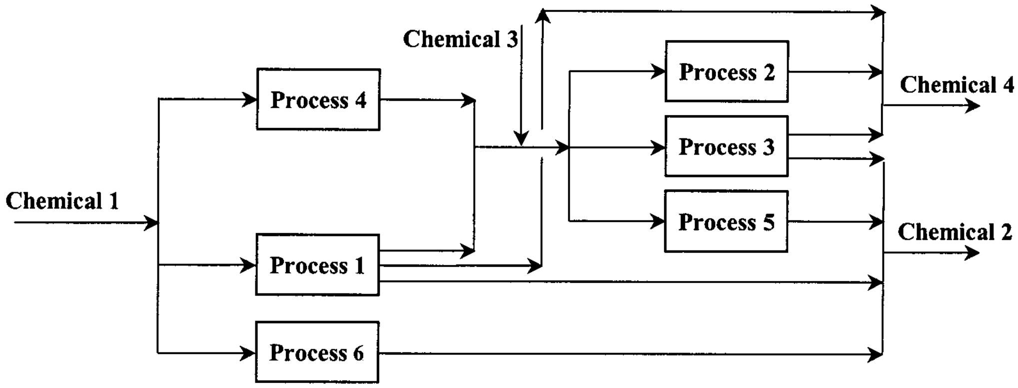

Figure 8. Network for example 2.

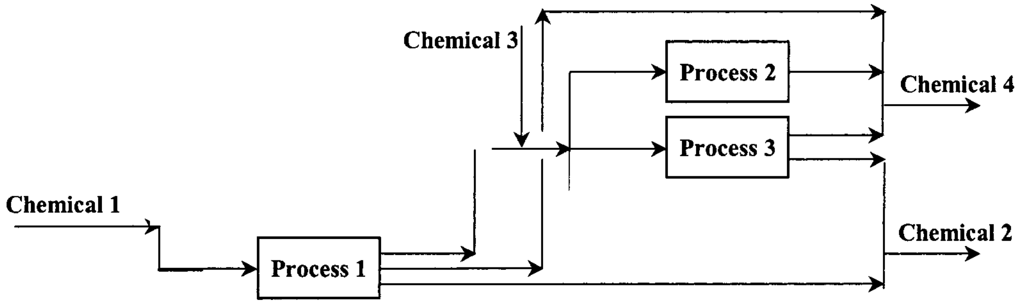

Figure 9. Solution network for example 2.

Table 7. NPVs for Restricted Region

λ_1/λ_2	z_1 ($\times 10^5$ \$)	z_2 ($\times 10^5$ \$)
1.0/0.0	9,285.33	13,452.70
0.9/0.1	9,284.73	13,454.59
0.8/0.2	9,284.13	13,456.49
0.7/0.3	9,283.54	13,458.39
0.6/0.4	9,282.94	13,460.29
0.5/0.5	9,282.34	13,462.18
0.4/0.6	9,281.75	13,464.08
0.3/0.7	9,281.15	13,465.98
0.2/0.8	9,280.56	13,467.87
0.1/0.9	9,279.96	13,469.77
0.0/1.0	9,279.36	13,471.67

of the nadir criterion vector. The new values for the ideal criterion vector are $z_1^* = \$\ 9{,}285.33 \times 10^5$ and $z_2^* = \$\ 13{,}471.67 \times 10^5$. All of these values are depicted in Figure 6.

Therefore, the aspiration levels constraints are added to the scalarized model AWTP, and the difference between the new values for the ideal and nadir criterion vectors is used for to scalarize the problem. Table 7 shows the values for the NPVs computed for the same λ values used, when the whole set of nondominated criterion vectors was computed.

As Figure 7 shows, a more concentrated number of points is obtained because of the reduced size of the region. Therefore, a procedure is proposed in which the interaction with the decision maker is based on his/her aspiration levels for each scenario. Starting at the first iteration, the whole set of nondominated criterion vectors is presented by the calculation of a finite number of points, and the aspiration levels for the first iteration are requested from the decision maker. The second iteration starts by reducing the set of nondominated criterion vectors to the region bounded from below by the aspiration levels supplied by the decision maker. A more concentrated number of points is now presented to the decision maker, and he/she can either pick a final

Table 8. Results for Deterministic Models for Example 2

	scenario 1			scenario 2		
	$Q_{1,t}/W_{1,t}$[a]	$Q_{2,t}/W_{2,t}$[b]	$Q_{3,t}/W_{3,t}$[c]	$Q_{1,t}/W_{1,t}$[a]	$Q_{2,t}/W_{2,t}$[b]	$Q_{3,t}/W_{3,t}$[c]
$t = 1$			55.7/52.0			57.1/57.1
$t = 2$	23.5/23.5		55.7/55.7	23.5/23.5		57.1/57.1
$t = 3$	23.5/23.5	21.3/21.3	55.7/46.1	40.5/40.5	17.2/17.2	57.1/51.1

[a] Process 1. [b] Process 2. [c] Process 3.

solution or supply new aspiration levels. The procedure continues in this fashion.

Example 2

A modification of example 1 that includes a new stream for producing chemical 4 directly from process 1 is now considered (Figure 8). The coefficient for the production of this chemical from chemical 1 is considered to be 1.0. Table 8 shows the result of solving the individual deterministic problems (SODPs) for the same two scenarios of example 1. Figure 9 shows the solution.

Notice that, in this case, only three expansions are required in the first scenario, whereas the second scenario requires four expansions. Moreover, different capacities are required in different periods. The components of the ideal and nadir criterion vectors are $z_1^* = \$\ 11{,}002.39 \times 10^5$, $z_2^* = \$\ 16{,}273.06 \times 10^5$ and $z_1^{\mathrm{nd}} = \$\ 10{,}824.72 \times 10^5$, $z_2^{\mathrm{nd}} = \$\ 15{,}272.88 \times 10^5$. Table 9 shows the NPVs calculated for a finite number of λ values using the scalarized AWTP. Figure 10 presents the feasible region and the whole set of nondominated criterion vectors. A discontinuity is observed in the line representing the set of nondominated criterion vectors and corresponds to the change from a planning solution requiring three expansions to one requiring four expansions. Moreover, a region consisting of nondominated unsupported criterion vectors is found below the dashed line. These vectors cannot be obtained using any set of probabilities in the original problem SOSP. The com-

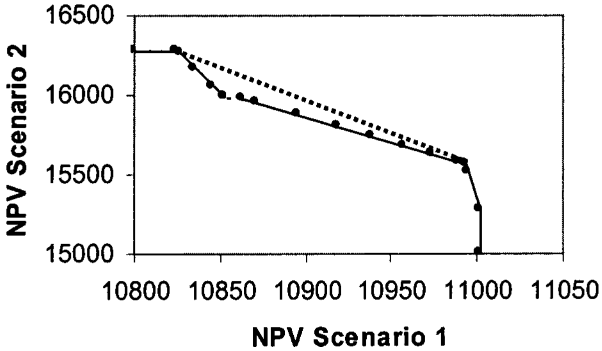

Figure 10. Scalarized multicriteria solution for example 2.

Table 9. NPVs for Scalarized AWTP Model

λ_1/λ_2	z_1 ($\times 10^5$ \$)	z_2 ($\times 10^5$ \$)
1.0/0.0	11,002.39	15,272.88
0.95/0.05	10,995.28	15,512.07
0.935/0.065	10,993.64	15,564.02
0.9/0.1	10,988.71	15,579.71
0.8/0.2	10,973.72	15,627.41
0.7/0.3	10,957.23	15,679.88
0.6/0.4	10,939.01	15,737.88
0.5/0.5	10,918.77	15,802.31
0.4/0.6	10,896.14	15,874.32
0.3/0.7	10,870.69	15,955.32
0.257/0.743	10,862.72	15,980.70
0.256/0.744	10,852.00	15,981.75
0.2/0.8	10,845.74	16,052.59
0.1/0.9	10,835.47	16,168.65
0.007/0.993	10,826.85	16,266.10
0.0/1.0	10,824.72	16,273.06

Table 10. Results for Deterministic Models for Example 3

	scenario 1			scenario 2			scenario 3		
period	$Q_{1,t}$[a]	$Q_{2,t}$[b]	$Q_{3,t}$[c]	$Q_{1,t}$[a]	$Q_{2,t}$[b]	$Q_{3,t}$[c]	$Q_{1,t}$[a]	$Q_{2,t}$[b]	$Q_{3,t}$[c]
$t = 1$			55.7			57.1			57.1
$t = 2$	23.5		55.7	23.5		57.1	23.5		57.1
$t = 3$	23.5	21.3	55.7	40.5	17.2	57.1	48.6	13.7	57.1

[a] Process 1. [b] Process 2. [c] Process 3.

putation of these nondominated criterion vectors was only possible because of the use of the augmented weighted Tchebycheff program.

Example 3

Example 2 was expanded to include one more scenario. Scenario 3 employs the parameters of scenario 1 increased by 50%. Once solved, the deterministic solutions are given in Table 10. Figure 11 presents the feasible region and the whole set of nondominated criterion vectors.

As one can see, when the dimensions are higher, one loses the ability to visualize the results, and therefore, one might need to rely on tables such as those presented here, the iterative procedure between the decision maker, and the use of the augmented weighted Tchebycheff program.

Conclusions

A procedure based on a scalarized augmented weighted Tchebycheff program was proposed to interact with a decision maker. The purpose is to find a compromised solution to the multiobjective MILP problem derived by a decomposition of a stochastic MILP model that considers multiperiod planning uncertainty through the concept of scenarios.

The advantages of this procedure are evident when comparisons with a point-estimate weighted-sums approach are made. The whole set of nondominated criterion vectors can be easily scanned because, for any value of the parameter λ, there is only one corresponding nondominated criterion vector. Moreover, occurrence of unsupported nondominated criterion vectors is readily considered, and discontinuities can be localized.

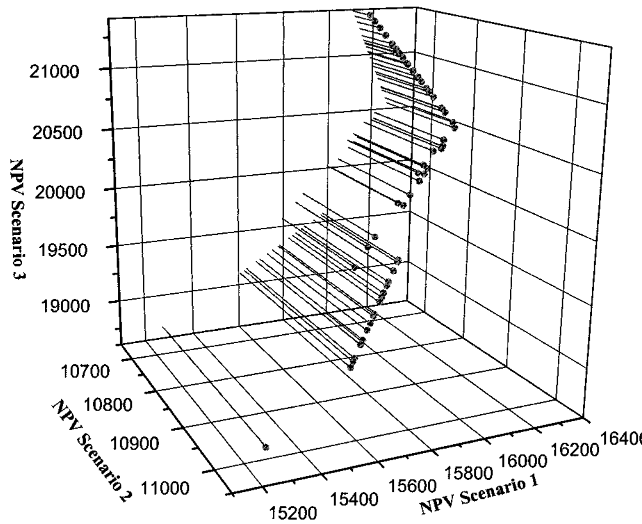

Figure 11. Multicriteria solution for example 3.

Nomenclature

Parameters

NC = number of chemicals
NP = number of processes
NM = number of markets
NT = number of periods

Variables

CI(t) = maximum investment allowed in period t
d^L_{jlt} = lower bound on sales (demand) of chemical j in market l within period t
d^U_{jlt} = upper bound on sales (demand) of chemical j in market l within period t
E_{it} = expansion capacity of plant i within period t
E^L_{it} = lower bound on the expansion capacity of plant i within period t
E^U_{it} = upper bound on the expansion capacity of plant i within period t
NEXP(i) = maximum number of expansions allowed for plant i
NPV = net present value
P_{jlt} = total purchases of chemical j in market l within period t
Q_{it} = capacity of plant i within period t
S_{jlt} = total sales of chemical j in market l within period t
W_{it} = operating capacity of plant i within period t
Y_{it} = binary variable for the realization of an expansion of plant i within period t
z_s = criterion vector defined as the net present value of each scenario
z_s^{**} = component of the nadir solution
z_s^{*} = component of the ideal solution

Greek Letters

α_{it} = investment cost coefficient of capacity expansion of plant i within period t

α^{L}_{jlt} = lower bound on purchases (availability) of chemical j in market l within period t

α^{U}_{jlt} = upper bound on purchases (availability) of chemical j in market l within period t

β_{it} = fixed investment cost coefficient of capacity expansion of plant i within period t

γ_{jlt} = sales price of chemical j in market l within period t

δ_{it} = operating cost coefficient of plant i within period t

Γ_{jlt} = purchase price of chemical j in market l within period t

η_{ij} = coefficient representing the amount of chemical j produced by plant i

μ_{ij} = coefficient representing the amount of chemical j produced by plant i

ρ_s = weight in the Tchebycheff program

Appendix

In this appendix, basic elements of multiobjective optimization theory are briefly outlined.

A-1. Formulation of Multiobjective Optimization. When there are p objectives to be optimized simultaneously, the mathematical problem can be formulated as follows

$$\max z_l = f_l(x) \qquad l = 1, 2, ..., p$$

$$\text{subject to}$$

$$g_i(x) \le b_i \qquad i = 1, ..., m$$

where x is a vector of n nonnegative real numbers.

A-2. Optimal and Efficient Points. A maximum (optimal) solution in the classical sense is one that attains the maximum value of all of the objectives simultaneously. The point x^* is optimal for the problem defined if and only if $x^* \in S$ and $f_l(x^*) \ge f_l(x)$ for all l and for all $x \in S$, where S is the feasible region.Thus, in general, there is no optimal solution to a multiobjective optimization problem. Therefore, one can be satisfied with obtaining only efficient solutions.

Definition. An efficient solution (also called a noninferior or Pareto optimal solution) is one in which no increase can be obtained in any of the objectives without causing a simultaneous decrease in at least one of the objectives. The solution x^* is efficient for the problem defined if and only if there exists no x in S such that

$$f_l(x) \ge f_l(x^*) \text{ for all } l \text{ and } f_l(x) > f_l(x^*) \text{ for at least one } l$$

This solution is obviously not unique. In most cases, the determination of the efficient set of solutions is not sufficient; one must choose an alternative that is by some definition "best". Let X_E be the set of efficient solutions. This set is called the *efficient frontier* or *Pareto optimal set*.

A slightly restricted definition of efficiency that eliminates efficient points of a certain anomalous type and gives a more satisfactory characterization is called "proper" efficiency. The point x^* is said to be a proper efficient solution if it is efficient and there exists a scalar $M > 0$ such that, for each l, $f_l(x) > f_l(x^*)$ and

$$\frac{f_l(x) - f_l(x^*)}{f_h(x^*) - f_h(x)} \le M \qquad l \ne h$$

for some h such that $f_h(x) < f_h(x^*)$.

Any efficient solution that is not properly efficient is called *improperly efficient*. Let X_{PE} be the set of properly efficient solutions. Then

$$X_{PE} \subseteq X_E$$

Question: Why exclude improperly efficient solutions?

Reason: Any rational decision maker would gladly give up an insignificant amount of objective h for an unbounded gain in objective l, unless they were totally satisfied with the amount of objective l they already had. However, this not the case because we assume that more is better.

Aspiration Levels. Objective function values that are satisfactory or desirable to the decision maker are called *aspiration levels* and denoted by $\bar{z}_i$, $i = 1, ..., p$. The vector $\bar{z} \in R^p$ consisting of aspiration levels is called a *reference point*.

A-3. Ranges of the Pareto Set. Next, we investigate the ranges of the set of Pareto optimal solutions. We assume that that the objective functions are bounded over the feasible region S. An objective vector maximizing each of the objective functions is called an ideal (or perfect) objective vector z^*. The components of the ideal objective vector z^* are obtained by solving the following p problems

$$\max f_l(x)$$

$$\text{subject to } x \in S$$

$$\text{for } l = 1, ..., p$$

The ideal objective vector generally is not feasible because there is some conflict among the objectives. Even though it can be considered a reference point. Note that in practice especially in the case of nonconvex problems the definition of the ideal vector assumes that we know the global minima of the individual objective functions, which is not that simple.

The lower bound vector of the Pareto optimal set is the so-called nadir objective vector, which can be estimated from a payoff table.

Definition. y^2 dominates y^1, written $y^2 > y^1$, if and only if $y^2 \in y^1 + D(y^1)$, where D is a *domination structure*. If D is a cone, we denote it by Λ and write

$$y^2 > y^1 \Rightarrow y^2 - y^1 \in \Lambda$$

Whereas S denotes the feasible region in decision space, Z denotes the feasible region in criterion space. In set theoretical notation

$$Z = \{z \in R^p: \ z = f(x), x \in S\}$$

Definition. Let $\bar{z}$ be in Z. $\bar{z}$ is nondominated if and only if there exists no z in Z such that $z \ge \bar{z}$ and $z \ne \bar{z}$, where $\ge$ is the component-wise order ($z \ge \bar{z} \Leftrightarrow z_i \ge \bar{z}_i$).

The set of all nondominated criterion vectors is called nondominated set N. Our interest in this set stems from the fact that the criterion vector cannot be optimal unless it is nondominated.

Let $Z^{\leq}$ be the convex hull of $[N \oplus \{z \in R^p: z \leq 0\}]$, where $\oplus$ means set addition.

Definition. Let $z \in N$. Then, if z is on the boundary of $Z^{\leq}$, z is called supported nondominated criterion vector. Otherwise, it is called an unsupported nondominated criterion vector.

Unsupported nondominated criterion vectors are dominated by some convex combination of other nondominated criterion vectors. Inverse images of supported nondominated criterion vectors are said to be supported efficient points (in decision space). Supported efficient solutions that are also proper are called supported efficient proper solutions.

A-4. Point-Estimate Weighted-Sums Approach. Next, we define the P_λ problem and describe the so-called point-estimate weighted-sums approach. By using this approach, we transform the original multiobjective optimization problem into a single-objective parametric optimization problem. The method is as follows: Each objective is multiplied by a strictly positive scalar λ_i. Then, the p weighted objectives are summed to form a weighted-sums objective function. Without loss of generality, we assume that each weighting vector $\lambda \in R^p$ is normalized so that its components sum to 1. By solving the following P_λ problem, one hopes that an optimal solution will be produced. Thus, the P_λ problem is

$$\text{(MOP)} \quad \begin{cases} \max \begin{pmatrix} f_1(x) \\ \vdots \\ f_p(x) \end{pmatrix} \\ \text{s.t.} \quad x \in S \end{cases} \Leftrightarrow$$

$$(\mathrm{P}_\lambda) \quad \begin{cases} \max \quad \lambda_1 f_1(x) + \ldots + \lambda_p f_p(x) \\ \text{s.t.} \quad x \in S \\ \sum_{i=1}^{p} \lambda_i = 1, \quad \lambda_i > 0 \end{cases}$$

Theorem 1 (Geoffrion). If x^* solves P_λ for a fixed $\lambda_i > 0$, $i = 1, ..., p$ then x^* is properly efficient.

What about the converse of theorem 1? (That is, if x^* is properly efficient, does it always solve P_λ for some $\lambda_i > 0$ such that $\sum_{i=1}^{p} \lambda_i = 1$?) Figure A-1 explains the case where the problem is nonconvex.

In this case, x^* is properly efficient, but there is no $\lambda > 0$ such that x^* solves P_λ. The problem is the nonconvexity of Z.

Theorem 2. If x^* is properly efficient, X is convex, and each f_i is concave, so that x^* solves P_λ for some $\lambda > 0$.

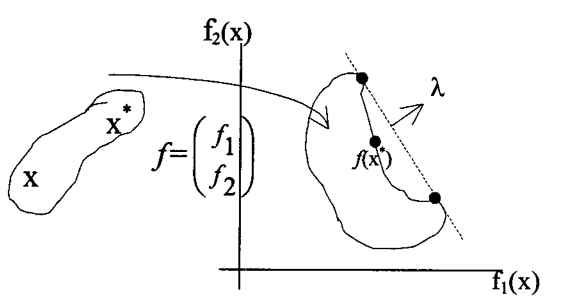

Figure A-1. Nonconvex case.

Corollary 1 (Combining Theorems 1 and 2). If X is *convex* and each f_i is *concave* on X, then x^* is properly efficient if and only if x^* solves P_λ for some $\lambda > 0$.

Therefore, we can use P_λ to generate all properly efficient solutions to convex vector optimization problems.

A-5. Multiparametric Decomposition. The following is the fundamental concept of multiparametric decomposition theory. In linear cases, the theory guarantees that a set of nondominated extreme points of the feasible region will be found. The form of the decomposition is as follows: Consider $F(\lambda,x) = \Sigma_{i\in I}\lambda_i f_i(x) = \lambda f(x)$, where λ is a weighting vector. Specifically, the proposed method exploits the following properties. Consider a linear programming problem P1 and a corresponding multiobjective problem P2 such that

$$\text{(P1)} \max F(x) = c^{\mathrm{T}}x \quad \text{s.t. } x \in X \qquad \text{(P2)} \max \{\lambda_1 f_1(x), ..., \lambda_k f_k(x)\} \quad \text{s.t. } x \in X$$

where X is the set of constraints $F(x) = \Sigma_i \lambda_i f_i(x)$ and $\Sigma_i \lambda_i = 1$.

The concepts behind the proposed method are as follows: The set of efficient extreme points of the decomposed problem is a subset of the set of extreme points of the original problem. Then, one possible procedure that solves a linear programming problem regarding the multiparametric decomposition algorithm is as follows: First, find $\{E_x\}$, the set of efficient extreme points, and then search for max $\{\Sigma f_i\}$ over E_x, which is the optimal solution to the original problem.

The multiparametric decomposition method expands the set of solutions by modifying the objective function and guarantees that an efficient solution of the original problem will be selected. This property provides the decision maker with a broad and flexible planning opportunity.

A-6. Tchebycheff Method. The Tchebycheff method has been designed to be user-friendly for the decision maker. Specifically, the distance from the ideal objective vector measured by a weighted Tchebycheff metric is minimized. Different solutions are obtained with different weighting vectors in the metric. The solution space is reduced by working with sequences of smaller and smaller subsets of the weighting vector spaces. The idea is to develop a sequence of progressively smaller subsets of the Pareto optimal set until the best compromise solution is found. At each different iteration, different objective vectors are presented to the decision maker, and (s)he is asked to select the most preferred solution. The feasible region is reduced, and new alternatives from the reduced space are presented to the decision maker for selection. However, there is a difference in the way weighting vectors are employed. Instead of using weighting vectors $\lambda \in \Lambda = \{\lambda \in R^p\colon \lambda_i > 0, \sum_{i=1}^{p} \lambda_i = 1\}$ as in the point-estimate weighted-sums approach, weighting vectors $\lambda \in \bar{\Lambda} = \{\lambda \in R^p\colon \lambda_i \geq 0, \sum_{i=1}^{p} \lambda_i = 1\}$ are used to define different Tchebycheff metrics. Therefore, the Tchebycheff method has the following advantages. (a) It can converge to nonextreme optimal solutions in linear multiobjective optimization. (b) The method can compute unsupported and improperly nondominated criterion vectors. This makes the method generalizable to integer and nonlinear multiobjective optimization. (c) The method uses conventional single-objective mathematical programming software.

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Received for review June 18, 2001
Revised manuscript received January 21, 2002
Accepted May 9, 2002

IE010530J